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Conformal anomaly of $(2,0)$ tensor multiplet in six dimensions and AdS/CFT correspondence

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Abstract: We compute the conformal anomaly in the free $d = 6$ superconformal (2,0) tensor multiplet theory on generic curved background. Up to a trivial covariant total-derivative term, it is given by the sum of the type A part proportional to the 6-d Euler density, and the type B part containing three independent conformal invariants. Multiplied by the factor $4N^3$, the latter Weyl-invariant part of the anomaly reproduces exactly the corresponding part of the conformal anomaly of large N multiple M5-brane (2,0) theory as predicted (hep-th/9806087) by AdS_7 supergravity on the basis of AdS/CFT correspondence. The coefficients of the type A anomaly differ by the factor $\frac{4}{7} \times 4N^3$, so that the free tensor multiplet anomaly does not vanish on a Ricci-flat background. The coefficient $4N^3$ is the same as found (hep-th/9703040) in the comparison of the tensor multiplet theory and the $d = 11$ supergravity predictions for the absorption cross-sections of gravitons by M5 branes, and in the comparison (hep-th/9911135) of 2- and 3-point stress tensor correlators of the free tensor multiplet with the AdS_7 supergravity predictions.

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Contents

1. Introduction and summary

While the low energy dynamics of a single M5 brane is described by the free $d = 6$, $\mathcal{N} = (2, 0)$ tensor multiplet, the low energy theory describing N coincident M5 branes remains rather mysterious. One of the key predictions of the supergravity description of multiple M5 branes is that the entropy [1] and the 2-point stress tensor correlators [2, 3] of the large-N theory should scale as N^3. Further quantitative information about this interacting $(2, 0)$ conformal theory can be obtained using the AdS/CFT correspondence [4, 5, 6]. In the large-N limit this leads directly to the analysis of $d = 11$ supergravity compactified on $AdS_7 \times S^4$. In particular, spectrum of the chiral operators, some of their 2- and 3-point functions and the structure of the anomalies have been studied [7]–[20].

In spite of the lack of a useful field-theoretic description of the large-N $(2, 0)$ theory, it is interesting to compare its properties to those of a $d = 6$ free conformal theory of a number $\sim N^3$ of tensor multiplets (after all, the free $(2, 0)$ tensor multiplet theory is the only $d = 6$ superconformal theory with the right symmetry properties which is known explicitly). The idea is to try to follow the pattern which worked in the case of the D3 brane theory where certain features of the strong coupling large-N $d = 4, \mathcal{N} = 4$ SYM theory as described by $AdS_5 \times S^5$ supergravity can be reproduced by a free theory of N^2 vector multiplets.

In a previous paper [21], we have found that the 2- and 3-point correlation functions of the stress tensor of $(2,0)$ theory as predicted by the $AdS_7 \times S^4$ supergravity [22, 15] are exactly the same as in the theory of $4N^3$ free tensor multiplets. The remarkable coefficient $4N^3$ is the same as found earlier in [2] in the comparison of the M5 brane world volume theory and the $d = 11$ supergravity expressions for the absorption cross-sections of longitudinally polarized gravitons by N M5 branes. This is not surprising since the ratios of the predictions for the 2-point stress tensor correlators and the absorption cross sections should be the same on the basis of unitarity [3, 5]. That the same coefficient appears also in the ratio of the 3-point correlators (which in general have a complicated structure parametrized by 3 independent constants [23]) is quite surprising and is likely to be a consequence of the extended $d = 6$ supersymmetry of the theory in question.

Here we extend such a comparison to conformal anomalies in external $d = 6$ metric. On the supergravity side of the AdS/CFT correspondence the conformal anomaly of the large-N M5 brane theory was already found in [11] (see also [24]). Below we compute the conformal anomaly of a free $d = 6$, $\mathcal{N} = (2,0)$ tensor multiplet which contains 5 scalars, 2 Weyl fermions and a chiral two-form.

In general, the trace anomaly in the stress tensor of a classically Weyl-invariant theory in $d = 2k$ dimensions has the following structure [25]–[29]: $\langle T\rangle = A + B + D$, where $A = aE_d$ is proprional to the Euler density in d dimensions (i.e. is a total derivative of a non-covariant expression), $B = \sum_n c_n I_n$ is a sum of independent Weyl invariants, i.e. Weyl tensor contractions with extra conformal derivative operators, $(C_{...})^k, \ldots, C_{...}(\nabla^{k-2} + \cdots)C_{...}$, and $D = \nabla_i J^i$ is a total derivative of a covariant expression. Only type-A and type-B anomalies are genuine (with the latter determining the UV related scale anomaly), while the type-D one is ambiguous (renormalization scheme dependent) as it can be changed by adding local covariant but not Weyl-invariant counterterms to the effective action. In 6 dimensions [27, 29]

$$\langle T\rangle = aE_6 + (c_1 I_1 + c_2 I_2 + c_3 I_3) + \nabla_i J^i\,, \tag{1.1}$$

where

$$E_6 = -\epsilon_6\epsilon_6 RRR\,, \qquad I_1 = C_{amnb}C^{mijn}C_i{}^{ab}{}_j\,, \qquad I_2 = C_{ab}{}^{mn}C_{mn}{}^{ij}C_{ij}{}^{ab}\,, \tag{1.2}$$

$$I_3 = C_{mabc}\Big(\nabla^2\delta^m_n + 4R^m_n - \frac{6}{5}R\delta^m_n\Big)C^{nabc} + \text{total derivative}\,. \tag{1.3}$$

Computing the conformal anomalies of the fields in the free tensor multiplet and comparing the resulting coefficients to the supergravity prediction [11] for the anomaly of the $(2,0)$ theory we have found that

$$a^{(2,0)} = \frac{16}{7}N^3\ a^{(\text{tens.})}\,, \qquad c_n^{(2,0)} = 4N^3\ c_n^{(\text{tens.})}\,. \tag{1.4}$$

Once again, the set of $4N^3$ tensor multiplets reproduces exactly the type-B or scale anomaly of the $(2,0)$ theory! However, the coefficients of the type-A anomaly then differ.

JHEP02(2000)013

The ratio $4N^3$ of the c_n coefficients is, in fact, in direct correspondence with the result for the ratio of the 2- and 3-point correlators of the stress tensor found in [21]. At the same time, the coefficient a of the Euler density term in the anomaly turns out to be related to a coefficient of a certain structure in the *4-point* correlation function of the stress tensor and should thus reflect some of the non-trivial dynamics of the interacting theory.[1]

To appreciate this novel feature of the $d = 6$ theory it is useful to compare the above results with what happens in the $d = 4$ case — the $\mathcal{N} = 4$ super Yang-Mills theory. In $d = 4$ the type-B anomaly contains just one independent term proportional to the square of the Weyl tensor whose coefficient is directly related to the one in the 2-point function $\langle TT \rangle$ of the stress tensor. The type-A (Euler) anomaly is instead related to the 3-point correlation function $\langle TTT \rangle$. Thus known non-renormalization theorems for the 2- and 3-point functions of the stress tensor multiplet guarantee that the trace anomaly of N^2 free $\mathcal{N} = 4$ vector multiplets should reproduce that of the full interacting non-abelian theory (see [30, 3]).

In $d = 6$ the coefficient a is related to the 4-point function and thus there is no reason to expect that it should not be renormalized.[2] It would be interesting to see if the R^4 correction to the $d = 11$ supergravity action generates an order N correction to the coefficient a in the supergravity expression for the conformal anomaly, like it does in the entropy of multiple M5 branes [32].

The above mentioned correspondence between particular terms in the conformal anomaly and correlation functions of stress tensor on flat background can be understood by studing the relation between the type-A and type-B conformal anomalies and corresponding terms in the effective action following [33, 28, 29, 34]. In a general even dimension $d = 2k$ the Weyl-invariant terms in type-B part of the conformal anomaly $(C_{...})^k$, ..., $C_{...}(\nabla^{k-2} + \cdots)C_{...}$ can be obtained by the Weyl variation from the non-local scale-dependent terms in the effective action like $\int (C_{...})^{k-1} \ln(\mu^{-d}\tilde{\Delta}_d) C_{...}$, ..., $\int C_{...}\tilde{\Delta}_d \ln(\mu^{-d}\tilde{\Delta}_d) C_{...}$. Here $\tilde{\Delta}_d = \nabla^d + \cdots$ is an appropriate 'Weyl-covariant' operator acting on Weyl tensor [33]. Expanded near flat space, $g_{mn} = \delta_{mn} + h_{mn}$, these terms start with $h^k, \ldots, h^2$, respectively, i.e. correspond to particular structures in the $k-, \ldots, 2-$ point correlators of the stress tensor in flat background, respectively.

In the case of $d = 4$, i.e. $k = 2$, the coefficient of the type-B anomaly (C^2) is thus

[1] Note that the type-A anomaly coefficient a (in any dimension) plays a special role from the point of view of the supergravity analysis [20].

[2] If the $d = 6$ free and interacting CFT's discussed above could be linked by a renormalization group flow preserving maximal supersymmetry, then our results would suggest that only the coefficient a of type-A anomaly can flow. However, it is difficult to see how such picture could be realized since the interacting theory at large-N does not have suitable scalar operators of dimensions $\Delta \leq 6$ which could be used to deform the theory (the only candidates break maximal supersymmetry). Similarly, the cohomological analysis of [31] indicates that a theory containing a free chiral two-form field cannot be continuously deformed in a non-trivial manner.

JHEP02(2000)013

correlated with the coefficient in the 2-point function $\langle TT\rangle$. In the case of $d=6$, i.e. $k=3$, the coefficient c_3 of the I_3 term in (1.1) corresponds to the one in the 2-point function, while the two other coefficients c_1, c_2 should be directly related to the two remaining independent coefficients in the generic $d=6$ CFT correlator $\langle TTT\rangle$.[3]

As for the type-A anomaly, the corresponding term in the effective action can be constructed by integrating the conformal anomaly like it was done in 2 [35] and 4 [36, 37] dimensions. One can introduce the modified Euler density [34] by combining the type-A anomaly with a particular type-D anomaly, $\tilde{E}_d = E_d + \nabla_i \tilde{J}^i$, where $\tilde{J}^i$ is a covariant expression such that the Weyl variation ($\delta g_{mn} = \phi g_{mn}$) of $\tilde{E}_d$ is proportional to $\Delta_d \phi$ where $\Delta_d = \nabla^d + \cdots$ is the Weyl-invariant operator acting on scalars (see, e.g., [38] and references there). Then the corresponding term in the effective action is [33] $\int \tilde{E}_d \frac{1}{\Delta_d} \tilde{E}_d$. Expanding this term near flat space and discarding local terms (which correspond to contact terms in the stress tensor correlators) it is possible to argue that the leading non-local structure with single ∂^{-2} pole contains $k+1$ graviton factors. Thus the type-A anomaly term is related to the $d/2+1$ point correlator of stress tensors, i.e. to 3-point correlator in $d=4$, 4-point correlator in $d=6$, etc.[4]

Coming back to (1.4), the disagreement of the total expressions for the conformal anomalies of the free tensor multiplet and interacting $(2,0)$ CFT can be easily seen using the results which already existed in the literature. Choosing a Ricci-flat $d=6$ background one finds that $(2,0)$ theory anomaly found in [11] vanishes, but the combined anomaly of the fields in the tensor multiplet is *non-zero*. For $R_{mn}=0$ the $d=6$ anomaly (1.1) depends (modulo a covariant total derivative term) only on 2 coefficients and can be written in the form [39]:[5] $\langle T\rangle = \frac{1}{(4\pi)^3}(s_1 E_6 + s_2 I_2)$. The coefficients s_1, s_2 are linear combinations of a, c_1, c_2, c_3 in (1.1). The coefficient s_2 is proportional to the graded number of degrees of freedom and thus vanishes ($5+3-2\times 4=0$) for the tensor multiplet. Computing the coefficient s_1 by combining the known results for a $d=6$ scalar [40, 41, 42], spinor [43] and the antisymmetric tensor [39] (using that the conformal anomaly of the chiral 2-form field is half of the anomaly of a non-chiral 2-form) one finds that (see [39, eqs. (4.21) and (4.39)]) $s_1 = -1/32\times 1/12$.[6] Thus, in contrast to the $d=4$ case where the conformal anomaly

[3] In a generic $d=6$ CFT the 3-point stress tensor correlator depends on 3 arbitrary parameters but one combination of them is related by Ward identity to the coefficient in the 2-point function [23].

[4] For example, in $d=4$ [33] $\Delta_4 \sim \partial^4 + \partial R_0 \partial + \cdots$, $R_0 \sim \partial\partial h$, $\tilde{E}_4 \sim E_4 + \partial^2 R_0$, so that $\int \tilde{E}_4 \frac{1}{\Delta_4} \tilde{E}_4 \sim \int R_0 \partial^{-2} \partial R_0 \partial \partial^2 R_0 + \cdots$ In $d=6$ $\tilde{E}_6$ has the structure (see [34]) $E_6 + R_0 \partial^2 R_0 + \partial^4 R_0$ and one finds that the first non-local term in $\int \tilde{E}_6 \frac{1}{\Delta_6} \tilde{E}_6$ is of order h^4.

[5] In the notation of [39] $E_6 = -32E$ with the Euler number being $\chi = \frac{1}{(4\pi)^3}\int d^6x\sqrt{g}\,\frac{2}{3}E$ and the invariants called I_1, I_2 in [39] are A_{16}, A_{17}, or, for $R_{mn}=0$, $I_2, -I_1$ in the present paper.

[6] It is easy to check that this anomaly cannot be cancelled by adding, e.g., a non-dynamical 5-form field which carries no degrees of freedom ($s_2=0$) but does produce a non-trivial conformal anomaly [44, 39] $s_1 = -1/32\times 2$. Duality rotation of scalars into 4-form fields also does not help ($s_1^{(4-\text{form})} - s_1^{(0-\text{form})} = 1/32\times 4/3$), and, in any case, the duality transformation is not consistent

of the $\mathcal{N} = 4$ vector multiplet vanishes on the Ricci flat background, there does not seem to exist a free $d = 6$ conformal matter theory[7] which shares the same property.[8]

Our aim below in section 2 will be to find the scalar, spinor and 2-form anomalies without assuming $R_{mn} = 0$ and thus to be able to compare the tensor multiplet anomaly to the supergravity prediction [11] for the anomaly of the $(2, 0)$ theory. Our starting point will be the general expression [41] for the corresponding Seeley coefficient b_6 (sometimes called also a_3) of the second-order laplacean. While the scalar field anomaly was computed in the past [41, 45, 47, 48] (though was not correctly put into the required conformal basis form), our explicit expressions for the spinor and the 2-form anomalies are new.

Appendix A contains conventions and definitions of the basic curvature invariants in $d = 6$. In appendix B we present the full expression for the Seeley coefficient b_6 taken from [41] but written in a slightly different form. In appendix C we give the results for the b_6 coefficients for the 1-form and 2-form fields in a space of general dimension d.

2. Conformal anomaly of $d = 6$ $(2, 0)$ tensor multiplet

The low-energy effective theory of a single M5-brane is described by the $(2, 0)$ tensor multiplet consisting of 5 scalars X^a, an antisymmetric tensor B_{ij} with (anti)selfdual strength and 2 Weyl fermions ψ^I_L. It is sufficient for the purposes of computing the conformal anomaly to consider the non-chiral $(2, 2)$ conformal model described by the following action

$$S = \int d^6x \sqrt{g}\Big(-\frac{1}{12}H^2_{ijk} - \frac{1}{2}\nabla_i X^\alpha \nabla^i X^\alpha - \frac{1}{10}X^\alpha X^\alpha R + i\bar{\psi}^I \gamma^m \nabla_m \psi^I\Big), \quad (2.1)$$

where $H_{ijk} = \partial_i B_{jk} + \partial_j B_{ki} + \partial_k B_{ij}$, $i, j, k = 1, \ldots, 6$, $\alpha = 1, \ldots, 10$, and $I = 1, 2$. The trace anomaly of the $(2, 0)$ tensor multiplet is then equal to $1/2$ of the trace anomaly of the $(2, 2)$ multiplet. Indeed, we can consistently disregard the gravitational anomalies of the $(2, 0)$ multiplet related to the imaginary part of the chiral 2-form and Weyl spinor determinants and focus only on their real part leading to the trace anomalies (see [49, 21]).[9] The anomaly of the $(2, 2)$ theory is given by the sum of trace anomalies of one non-chiral 2-form, 10 conformal scalar fields and 2 Dirac fermions which we shall compute separately below.

with conformal invariance.

[7] Note, however, that the combined Seeley coefficient b_6 of the fields of the $d = 11$ supergravity or its reduction to 6 dimensions does vanish [39].

[8] Possible importance of theories in which the coefficients a and c_i are related in such a way that the anomaly vanishes for $R_{mn} = 0$ was advocated in [46].

[9] While the definition of the partition function of a chiral p-form theory on generic manifolds is subtle (see [50, 51]) the coefficients in the local trace anomaly are universal (cannot depend on details of space-time topology) and can be determined, e.g., by a Feynman diagram calculation near flat space-time.

2.1 Conformal anomaly as Seeley-DeWitt coefficient

We begin by recalling the relation of free-theory conformal anomaly to the Seeley-DeWitt coefficients. Consider a one-loop approximation to a model of a bosonic field ϕ taking values in a smooth vector bundle V over a compact smooth riemannian manifold M of dimension d. The partition function and the effective action of the model are given by

$$Z = \int d\phi \, \mathrm{e}^{-\frac{1}{2}\int_M \phi\Delta\phi}, \qquad \Gamma = \frac{1}{2}\log\det\,\Delta\,, \tag{2.2}$$

$$\Delta = -\nabla^2 - E\,, \tag{2.3}$$

where ∇ is a covariant derivative on V and the matrix function E is an endomorphism of V. It is well-known that the trace anomaly is related to the logarithmically divergent part of the effective action. Using the Seeley-DeWitt asymptotic expansion [52, 53]

$$(\mathrm{Tr}\,\mathrm{e}^{-s\Delta})_{s\to 0} \sim \sum_{p=0}^{d} s^{\frac{1}{2}(p-d)} \int_M d^dx\,\sqrt{g}\,b_p$$

we get the logarithmically divergent term in the effective action

$$\Gamma_\infty = -\frac{1}{2}\log\frac{L^2}{\mu^2}\int_M d^dx\,\sqrt{g}\,b_d\,,$$

where $L\to\infty$ is an UV cut-off. The trace anomaly of the stress tensor is then equal to b_d:

$$\langle T_m{}^m(x)\rangle = b_d(x)\,. \tag{2.4}$$

Thus to find the conformal anomaly of the $d=6$ tensor multiplet we need to know the coefficients b_6 for various second-order Laplace operators corresponding to the fields in (2.1).

The coefficient b_6 was explicitly computed for any operator of the form (2.3) in [41], and can be written as follows

$$\begin{aligned} b_6(\Delta) = \frac{1}{(4\pi)^3 7!}\mathrm{tr}_V\Big[& 18A_1 + 17A_2 - 2A_3 - 4A_4 + 9A_5 + \\ & + 28A_6 - 8A_7 + 24A_8 + 12A_9 + \frac{35}{9}A_{10} - \frac{14}{3}A_{11} + \frac{14}{3}A_{12} - \\ & - \frac{208}{9}A_{13} + \frac{64}{3}A_{14} - \frac{16}{3}A_{15} + \frac{44}{9}A_{16} + \frac{80}{9}A_{17} + \\ & + 14\Big(8V_1 + 2V_2 + 12V_3 - 12V_4 + 6V_5 - 4V_6 + 5V_7 + 6V_8 + \\ & \quad + 60V_9 + 30V_{10} + 60V_{11} + 30V_{12} + 10V_{13} + 4V_{14} + \\ & \quad + 12V_{15} + 30V_{16} + 12V_{17} + 5V_{18} - 2V_{19} + 2V_{20}\Big)\Big], \end{aligned} \tag{2.5}$$

where the invariants A_s and V_p (depending on the metric tensor, the connection curvature tensor and the endomorphism E) are listed in the appendix A. An explicit expression for b_6 can be found in appendix B.

As already discussed in the Introduction, the conformal anomaly in a classically Weyl-invariant $d=6$ theory, or the coefficient b_6 for a conformally invariant kinetic operator, must have the form [27, 28, 29]

$$b_6 = aE_6 + c_1 I_1 + c_2 I_2 + c_3 I_3 + \nabla_i J^i \, . \tag{2.6}$$

Here the first term is the type-A anomaly proportional to the Euler density polynomial

$$\begin{aligned} E_6 &= -\epsilon_{m_1 n_1 m_2 n_2 m_3 n_3} \epsilon^{a_1 b_1 a_2 b_2 a_3 b_3} R^{m_1 n_1}_{a_1 b_1} R^{m_2 n_2}_{a_2 b_2} R^{m_3 n_3}_{a_3 b_3} \\ &= -8A_{10} + 96A_{11} - 24A_{12} - 128A_{13} - 192A_{14} + 192A_{15} - 32A_{16} + 64A_{17} \, , \end{aligned} \tag{2.7}$$

while the next three terms represent the type-B anomalies that are combinations of the following Weyl invariants

$$\begin{aligned} I_1 &= C_{amnb} C^{mijn} C_i{}^{ab}{}_j \\ &= \frac{19}{800}A_{10} - \frac{57}{160}A_{11} + \frac{3}{40}A_{12} + \frac{7}{16}A_{13} + \frac{9}{8}A_{14} - \frac{3}{4}A_{15} - A_{17} \, , \end{aligned} \tag{2.8}$$

$$\begin{aligned} I_2 &= C_{ab}{}^{mn} C_{mn}{}^{ij} C_{ij}{}^{ab} \\ &= \frac{9}{200}A_{10} - \frac{27}{40}A_{11} + \frac{3}{10}A_{12} + \frac{5}{4}A_{13} + \frac{3}{2}A_{14} - 3A_{15} + A_{16} \, , \end{aligned} \tag{2.9}$$

$$\begin{aligned} I_3 &= C_{mabc} \left(\nabla^2 \delta^m_n + 4R^m_n - \frac{6}{5} R \delta^m_n \right) C^{nabc} + \nabla_i \mathcal{J}^i \\ &= -\frac{11}{50}A_{10} + \frac{27}{10}A_{11} - \frac{6}{5}A_{12} - 3A_{13} - 4A_{14} + 4A_{15} + \frac{1}{10}A_6 - A_7 + A_9 + \nabla_i \mathcal{J}^i \, , \end{aligned} \tag{2.10}$$

where

$$C_{abcd} = R_{abcd} - \frac{1}{4}(g_{ac}R_{bd} + g_{bd}R_{ac} - g_{ad}R_{bc} - g_{bc}R_{ad}) + \frac{1}{20}(g_{ac}g_{bd} - g_{ad}g_{bc})R \tag{2.11}$$

is the Weyl tensor in 6 dimensions,

$$\begin{aligned} \nabla_i \mathcal{J}^i = 5C_5 - 8C_7 &= 3A_3 - 6A_4 + 3A_5 + \frac{1}{2}A_6 - 5A_7 + \\ &+ 5A_9 + 2A_{13} - 2A_{14} - 4A_{15} + 2A_{16} + 8A_{17} \, , \end{aligned}$$

and the invariants C_k are defined in appendix A. The invariant I_3 was defined up to the total derivative term $\nabla_i \mathcal{J}^i$ in [34] (similar invariants in [27, 28, 38] are linear combinations of I_3 with the other invariants), and is related to the invariant Ω_6 used in [47] as $I_3 = 3\Omega_6 + 16I_1 - 4I_2$. Finally, the last term in eq. (2.6) is a total derivative of a covariant expression which can be cancelled by the Weyl variation of a finite local covariant counterterm. Thus, only the coefficients of the first four terms in (2.6) have unambiguous (scheme-independent) meaning and will be of our main interest below.

JHEP02(2000)013

2.2 Conformal anomaly of a scalar field

In the simplest case of a $d = 6$ conformal scalar field the Laplace operator Δ is given by

$$\Delta_S = -\nabla^2 + \frac{1}{5}R, \tag{2.12}$$

where the connection in ∇ is trivial ($F_{ij} = 0$), and the endomorphism E is

$$E = -\frac{1}{5}R.$$

A straightforward calculation based on (2.5) gives the trace anomaly of the conformal scalar as

$$\begin{aligned}\mathcal{A}_S &= \langle T_m^{Sm}(x)\rangle = b_6^S(x) = \\ &= \frac{1}{(4\pi)^3\, 7!}\Big(\frac{6}{5}A_1 + \frac{1}{5}A_2 - 2A_3 - 4A_4 + 9A_5 - 8A_7 + \\ &\qquad + \frac{8}{5}A_8 + 12A_9 - \frac{7}{225}A_{10} + \frac{14}{15}A_{11} - \frac{14}{15}A_{12} - \frac{32}{45}A_{13} - \\ &\qquad - \frac{16}{15}A_{14} - \frac{16}{3}A_{15} + \frac{44}{9}A_{16} + \frac{80}{9}A_{17}\Big)\,.\end{aligned} \tag{2.13}$$

This formula can be rewritten in the form (2.6) by using the identities from appendix A:

$$\mathcal{A}_S = \frac{1}{(4\pi)^3\, 7!}\Big(-\frac{5}{72}E_6 - \frac{28}{3}I_1 + \frac{5}{3}I_2 + 2I_3 + \nabla_i J^i\Big)\,, \tag{2.14}$$

where

$$\nabla_i J^i = \frac{6}{5}C_1 - \frac{2}{5}C_2 + 4C_3 + \frac{12}{5}C_4 + \frac{17}{5}C_6 + 12C_7\,.$$

Note that our expression (2.14) differs from the one derived in [47].

2.3 Conformal anomaly of a Dirac fermion

The square of the Dirac operator gives the following second-order differential operator Δ_F

$$\Delta_F = -(\nabla\!\!\!/)^2 = -\nabla^2 + \frac{1}{4}R\cdot 1\,. \tag{2.15}$$

The connection in ∇ in this case is nontrivial with

$$F_{ij} = \frac{1}{4}R_{ijab}\gamma^{ab}$$

and the endomorphism E is

$$E = -\frac{1}{4}R\cdot 1\,.$$

The calculation of the corresponding b_6 coefficient (2.5) gives the following expression for the trace anomaly of a Dirac fermion (we account for the Fermi statistics by reversing the sign of b_6)

$$\begin{aligned}\mathcal{A}_F &= \langle T^{Fm}_m(x)\rangle = -b^F_6(x) = \\ &= -\frac{8}{(4\pi)^3\,7!}\Big(-3A_1 + \frac{5}{4}A_2 - 9A_3 + 3A_4 - 5A_5 + \frac{7}{2}A_6 - 8A_7 - \\ &\qquad - 4A_8 - 9A_9 - \frac{35}{72}A_{10} + \frac{7}{3}A_{11} + \frac{49}{24}A_{12} + \frac{44}{9}A_{13} - \\ &\qquad - \frac{20}{3}A_{14} + \frac{5}{3}A_{15} - \frac{101}{18}A_{16} - \frac{109}{9}A_{17}\Big)\,. \end{aligned} \tag{2.16}$$

By using the identities from appendix A we can rewrite this in the form (2.6)

$$\mathcal{A}_F = \frac{1}{(4\pi)^3\,7!}\Big(-\frac{191}{72}E_6 - \frac{896}{3}I_1 - 32I_2 + 40I_3 + \nabla_i J^i\Big)\,, \tag{2.17}$$

where

$$\nabla_i J^i = 24C_1 - \frac{148}{15}C_2 + 136C_3 + 48C_4 - 168C_5 + 96C_6 + 352C_7\,.$$

2.4 Conformal anomaly of a 2-form field

To find the conformal anomaly of an antisymmetric tensor field we use a covariant gauge fixing with the standard triangle-like ghost structure [54]. This leads to the following representation for the partition function

$$Z_{(2)} = (\det\Delta^{(2)})^{-1/2}\det\Delta^{(1)}(\det\Delta^{(0)})^{-3/2}\,, \tag{2.18}$$

where the Hodge-De Rham operators $\Delta^{(p)}$ are defined as

$$\begin{aligned}(\Delta^{(2)})^{ab}_{mn} &= -\nabla^2\delta^{ab}_{mn} + 2R^{[a}_{[m}\delta^{b]}_{n]} - R_{mn}{}^{ab}\,, \\ (\Delta^{(1)})^n_m &= -\nabla^2\delta^n_m + R^n_m\,, \\ \Delta^{(0)} &= -\nabla^2\,.\end{aligned}$$

As follows from (2.18), the conformal anomaly of a 2-form field B_{mn} is given by

$$\mathcal{A}_B = b^{(2)}_6 - 2b^{(1)}_6 + 3b^{(0)}_6\,, \tag{2.19}$$

where $b^{(p)}_6$ are the Seeley-DeWitt coefficients of the operators $\Delta^{(p)}$.

The coefficient $b^{(0)}_6$ is obtained from (2.5) by dropping out all the invariants V_p (in this case of $\Delta^{(0)}$ the connection and E are trivial)

$$\begin{aligned}b^{(0)}_6 &= \frac{1}{(4\pi)^3 7!}\Big(18A_1 + 17A_2 - 2A_3 - 4A_4 + 9A_5 + 28A_6 - \\ &\qquad - 8A_7 + 24A_8 + 12A_9 + \frac{35}{9}A_{10} - \frac{14}{3}A_{11} + \frac{14}{3}A_{12} - \\ &\qquad - \frac{208}{9}A_{13} + \frac{64}{3}A_{14} - \frac{16}{3}A_{15} + \frac{44}{9}A_{16} + \frac{80}{9}A_{17}\Big)\,. \end{aligned} \tag{2.20}$$

The coefficient $b_6^{(1)}$ of the Hodge-De Rham operator $\Delta^{(1)}$ acting on 1-forms is found by taking into account that the connection is defined by the Christoffel symbols so that

$$(F_{ij})_a{}^b = R_{ija}{}^b\,,$$

while the endomorphism E is

$$E_a{}^b = -R_a{}^b\,.$$

Computing the invariants V_p and expressing them in terms of A_s, we get for $d = 6$ [10]

$$\begin{aligned} b_6^{(1)}(x) = \frac{1}{(4\pi)^3\,7!}\Big(&24A_1 - 66A_2 + 352A_3 + 32A_4 - 58A_5 - 140A_6 + 792A_7 + \\ &+ 32A_8 - 96A_9 - \frac{140}{3}A_{10} + 420A_{11} - 70A_{12} - \frac{2600}{3}A_{13} + \\ &+ 16A_{14} + 444A_{15} - \frac{164}{3}A_{16} - \frac{344}{3}A_{17}\Big). \end{aligned} \tag{2.21}$$

To compute the coefficient $b_6^{(2)}$ of the operator $\Delta^{(2)}$ acting on 2-forms, we note that the curvature tensor of the connection and the endomorphism here are

$$\begin{aligned} (F_{ij})_{ab}^{cd} &= 2R_{ij[a}{}^{[c}\delta_{b]}^{d]}\,, \\ E_{ab}^{cd} &= -2R_{[a}^{[c}\delta_{b]}^{d]} + R_{ab}{}^{cd}\,. \end{aligned}$$

By using the formulas for the traces from appendix C, we find in $d = 6$

$$\begin{aligned} b_6^{(2)}(x) = \frac{1}{(4\pi)^3\,7!}\Big(&-66A_1 + 3A_2 - 254A_3 + 164A_4 + 107A_5 + 28A_6 - 120A_7 - \\ &- 88A_8 + 348A_9 + \frac{595}{3}A_{10} - 2478A_{11} + 518A_{12} + \frac{10384}{3}A_{13} + \\ &+ 4912A_{14} - 4896A_{15} + \frac{2992}{3}A_{16} - \frac{1616}{3}A_{17}\Big)\,. \end{aligned} \tag{2.22}$$

Then from (2.19) we obtain the conformal anomaly of the 2-form field B_{ij}

$$\begin{aligned} \mathcal{A}_B &= \langle T_m^{Bm}(x)\rangle \\ &= \frac{1}{(4\pi)^3\,7!}\Big(-60A_1 + 186A_2 - 964A_3 + 88A_4 + 250A_5 + 392A_6 - 1728A_7 - \\ &\qquad - 80A_8 + 576A_9 + \frac{910}{3}A_{10} - 3332A_{11} + 672A_{12} + \frac{15376}{3}A_{13} + \\ &\qquad + 4944A_{14} - 5800A_{15} + \frac{3364}{3}A_{16} - \frac{848}{3}A_{17}\Big)\,. \end{aligned} \tag{2.23}$$

One can easily check that on a Ricci flat manifold the anomaly coincides with the one found in [39].

[10] We present the coefficients $b_6^{(1)}$ and $b_6^{(2)}$ for generic dimension d of the manifold in appendix C.

The identities from appendix A allow to represent (2.23) in the required form (2.6)

$$\mathcal{A}_B = \frac{1}{(4\pi)^3\,7!}\Big(-\frac{221}{4}E_6 - \frac{8008}{3}I_1 - \frac{2378}{3}I_2 + 180I_3 + \nabla_i J^i\Big)\,, \tag{2.24}$$

where

$$\nabla_i J^i = -60C_1 + \frac{2036}{15}C_2 - 1152C_3 - 120C_4 - 504C_5 - 646C_6 + 856C_7\,.$$

2.5 Conformal anomaly of the free $(2,0)$ tensor multiplet

Finally, all is prepared to write down the expression for the conformal anomaly of the chiral $(2,0)$ tensor multiplet

$$\begin{aligned}\mathcal{A}_{tens.} &= \frac{1}{2}\left(\mathcal{A}_B + 10\mathcal{A}_S + 2\mathcal{A}_F\right) \qquad (2.25)\\ &= \frac{1}{(4\pi)^3\,7!}\Big(84A_2 - 420A_3 + 210A_5 + 168A_6 - 840A_7 + 420A_9 + \frac{777}{5}A_{10} -\\ &\qquad - 1680A_{11} + 315A_{12} + 2520A_{13} + 2520A_{14} - 2940A_{15} + 630A_{16}\Big)\,,\end{aligned}$$

or, in the form (2.6),

$$\mathcal{A}_{tens.} = \frac{1}{(4\pi)^3\,7!}\Big(-\frac{245}{8}E_6 - 1680I_1 - 420I_2 + 140I_3 + \nabla_i J^i\Big)\,, \tag{2.26}$$

where

$$\nabla_i J^i = 56C_2 - 420C_3 - 420C_5 - 210C_6 + 840C_7\,.$$

It is easy to see using the identities in appendix A that for $R_{mn} = 0$ this expression agrees with the expression following from [39] which was already mentioned in the introduction, i.e. $\mathcal{A}_{\rm tens.} = -\frac{1}{(4\pi)^3\times 32\times 12}E_6$, up to a covariant total derivative term ($\sim C_5 = \frac{1}{2}\nabla^2(C^2_{mnkl})$).[11]

Let us now compare the result (2.26) with the conformal anomaly of the interacting $(2,0)$ theory describing large number N of coincident M5 branes as predicted on the basis of AdS/CFT correspondence in [11]. In terms of the invariants we are using here the expression obtained in [11] takes the form (note that it vanishes for $R_{mn} = 0$ as it should)

$$\mathcal{A}_{(2,0)} = \frac{4N^3}{(4\pi)^3\,7!}\Big(-\frac{35}{2}E_6 - 1680I_1 - 420I_2 + 140I_3 + \nabla_i J^i\Big)\,, \tag{2.27}$$

where

$$\nabla_i J^i = 420C_3 - 504C_4 - 840C_5 - 84C_6 + 1680C_7\,.$$

[11] Note that for $R_{mn} = 0$ and ignoring the total derivative term one has the following relations $I_3 = -I_2 + 4I_1\,,\ \ E_6 = -32I_2 - 64I_1\,.$

Comparing (2.26) and (2.27) we conclude that up to the common factor $4N^3$ only the coefficient in front of the Euler polynomial is different (the difference in coefficients of total derivative terms is not important since they are scheme-dependent). The interpretation of this result was already discussed in the introduction.

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A. Conventions, invariants and identities

We use the following conventions for the curvature tensors:

$$[\nabla_a, \nabla_b]V^c = R_{ab}{}^c{}_d V^d\,, \qquad R_{ab} = R_{ca}{}^c{}_b\,, \qquad R = R^a_a\,, \qquad [\nabla_a, \nabla_b]\phi = F_{ab}\phi\,.$$

The basis of metric invariants is[12]

$$\begin{aligned}
A_1 &= \nabla^4 R & A_2 &= (\nabla_a R)^2 & A_3 &= (\nabla_a R_{mn})^2 \\
A_4 &= \nabla_a R_{bm} \nabla^b R^{am} & A_5 &= (\nabla_a R_{mnij})^2 & A_6 &= R\nabla^2 R \\
A_7 &= R_{ab}\nabla^2 R^{ab} & A_8 &= R_{ab}\nabla_m \nabla^b R^{am} & A_9 &= R_{abmn}\nabla^2 R^{abmn} \\
A_{10} &= R^3 & A_{11} &= RR_{ab}^2 & A_{12} &= RR_{abmn}^2 \\
A_{13} &= R_a{}^m R_m{}^i R_i{}^a & A_{14} &= R_{ab}R_{mn}R^{ambn} & A_{15} &= R_{ab}R^{amnl}R^b{}_{mnl} \\
A_{16} &= R_{ab}{}^{mn} R_{mn}{}^{ij} R_{ij}{}^{ab} & A_{17} &= R_{ambn}R^{aibj}R^m{}_i{}^n{}_j\,.
\end{aligned}$$

Another convenient basis of the metric invariants is obtained by replacing the Ricci tensor R_{ij} by its traceless part

$$B_{ij} = R_{ij} - \frac{1}{d} R g_{ij}\,,$$

and the Riemann tensor — by the Weyl tensor

$$C_{ijkl} = R_{ijkl} - \frac{1}{d-2}(g_{jl}B_{ik} - g_{jk}B_{il} + g_{ik}B_{jl} - g_{il}B_{jk}) - \frac{R}{d(d-1)}(g_{jl}g_{ik} - g_{jk}g_{il})\,.$$

[12] We use the same notation as in [47]. Note, however, that there are a number of misprints in that paper.

JHEP02(2000)013

Then we get the following 17 invariants B_s

$$
\begin{array}{lll}
B_1 = \nabla^4 R & B_2 = (\nabla_a R)^2 & B_3 = (\nabla_a B_{mn})^2 \\
B_4 = \nabla_a B_{bm} \nabla^b B^{am} & B_5 = (\nabla_a C_{mnij})^2 & B_6 = R \nabla^2 R \\
B_7 = B_{ab} \nabla^2 B^{ab} & B_8 = B_{ab} \nabla_m \nabla^b B^{am} & B_9 = C_{abmn} \nabla^2 C^{abmn} \\
B_{10} = R^3 & B_{11} = R B_{ab}^2 & B_{12} = R C_{abmn}^2 \\
B_{13} = B_a{}^m B_m{}^i B_i{}^a & B_{14} = B_{ab} B_{mn} C^{ambn} & B_{15} = B_{ab} C^{amnl} C^b{}_{mnl} \\
B_{16} = C_{ab}{}^{mn} C_{mn}{}^{ij} C_{ij}{}^{ab} & B_{17} = C_{ambn} C^{aibj} C^m{}_i{}^n{}_j \, . &
\end{array}
$$

The invariants A_s are related to B_s as follows

$$
\begin{aligned}
A_1 &= B_1 \,, \qquad A_2 = B_2 \,, \qquad A_3 = B_3 + \frac{1}{d} B_2 \,, \qquad A_4 = B_4 + \frac{d-1}{d^2} B_2 \\
A_5 &= B_5 + \frac{4}{d-2} B_3 + \frac{2}{d(d-1)} B_2 \,, \qquad A_6 = B_6 \,, \qquad A_7 = B_7 + \frac{1}{d} B_6 \\
A_8 &= \frac{d}{d-2} B_8 + \frac{1}{2d} B_6 + \frac{2}{d-2} B_{14} - \frac{2d}{(d-2)^2} B_{13} - \frac{2}{(d-1)(d-2)} B_{11} \\
A_9 &= B_9 + \frac{4}{d-2} B_7 + \frac{2}{d(d-1)} B_6 \,, \qquad A_{10} = B_{10} \\
A_{11} &= B_{11} + \frac{1}{d} B_{10} \,, \qquad A_{12} = B_{12} + \frac{4}{d-2} B_{11} + \frac{2}{d(d-1)} B_{10} \\
A_{13} &= B_{13} + \frac{3}{d} B_{11} + \frac{1}{d^2} B_{10} \\
A_{14} &= B_{14} - \frac{2}{d-2} B_{13} + \frac{2d-3}{d(d-1)} B_{11} + \frac{1}{d^2} B_{10} \\
A_{15} &= B_{15} + \frac{4}{d-2} B_{14} + \frac{2(d-4)}{(d-2)^2} B_{13} + \frac{1}{d} B_{12} + \frac{4(2d-3)}{d(d-1)(d-2)} B_{11} + \frac{2}{d^2(d-1)} B_{10} \\
A_{16} &= B_{16} + \frac{12}{d-2} B_{15} + \frac{24}{(d-2)^2} B_{14} + \frac{8(d-4)}{(d-2)^3} B_{13} + \\
&\quad + \frac{6}{d(d-1)} B_{12} + \frac{24}{d(d-1)(d-2)} B_{11} + \frac{4}{d^2(d-1)^2} B_{10} \\
A_{17} &= B_{17} - \frac{3}{d-2} B_{15} + \frac{3(d-4)}{(d-2)^2} B_{14} + \frac{2(8-3d)}{(d-2)^3} B_{13} - \\
&\quad - \frac{3}{2d(d-1)} B_{12} + \frac{3(d-4)}{d(d-1)(d-2)} B_{11} + \frac{d-2}{d^2(d-1)^2} B_{10} \, .
\end{aligned}
$$

One can show that the following linear combinations

$$
\begin{aligned}
C_1 &= B_1 \,, \qquad C_2 = B_2 + B_6 \,, \qquad C_3 = B_3 + B_7 \\
C_4 &= B_4 + B_8 \,, \qquad C_5 = B_5 + B_9 \\
C_6 &= \frac{(d-2)^2}{4d^2} B_2 - B_4 - \frac{1}{d-1} B_{11} - \frac{d}{d-2} B_{13} + B_{14} \\
C_7 &= \frac{(d-2)(d-3)}{4d^2(d-1)} B_2 - \frac{d-3}{d-2}(B_3 - B_4) + \frac{1}{4} B_5 + \frac{1}{2d} B_{12} + \frac{1}{2} B_{15} - \frac{1}{4} B_{16} - B_{17}
\end{aligned}
$$

are total derivatives. These are the important identities used in the main text.

The basis of invariants V_p depending on the curvature F_{ij} and the endomorphism E is

$$
\begin{array}{lll}
V_1 = \nabla_k F_{ij} \nabla^k F^{ij} & V_2 = \nabla_j F_{ij} \nabla^k F^{ik} & V_3 = F_{ij} \nabla^2 F^{ij} \\
V_4 = F_{ij} F^{jk} F_k{}^i & V_5 = R_{mnij} F^{mn} F^{ij} & V_6 = R_{jk} F^{jn} F^k{}_n \\
V_7 = R F_{ij} F^{ij} & V_8 = \nabla^4 E & V_9 = E \nabla^2 E \\
V_{10} = \nabla_k E \nabla^k E & V_{11} = V^3 & V_{12} = E F_{ij}^2 \\
V_{13} = R \nabla^2 E & V_{14} = R_{ij} \nabla^i \nabla^j E & V_{15} = \nabla_k R \nabla^k E \\
V_{16} = E E R & V_{17} = E \nabla^2 R & V_{18} = E R^2 \\
V_{19} = E R_{ij}^2 & V_{20} = E R_{ijkl}^2 . &
\end{array}
$$

B. Heat kernel expansion and b_6 coefficient

The heat kernel coefficients for a general Laplace operator of the form $\Delta = -\nabla^2 - E$ with connection of curvature F_{ab} as defined in appendix A and matrix potential E, were computed, up to and including b_6, in [41]. For convenience, we present the explicit form of these leading terms in the heat kernel expansion below and use this opportunity to cast this expansion into a form that may be advantageous for certain computational purposes. In principle, one can compute various terms of the heat kernel expansion by using the standard perturbation theory for a quantum mechanical path integral [55, 56]. The latter naturally separates connected and disconnected particle theory diagrams and suggests the following representation of the heat kernel expansion

$$
\mathrm{Tr}\left[\sigma(x) e^{-s\Delta}\right] = \frac{1}{(4\pi s)^{d/2}} \mathrm{Tr}\left[\sigma(x) \sum_{n=0}^{\infty} a_{2n} s^n\right] = \frac{1}{(4\pi s)^{d/2}} \mathrm{Tr}\left[\sigma(x) \exp\left(\sum_{n=1}^{\infty} \alpha_{2n} s^n\right)\right].
$$

The standard b_{2n} coefficients are then

$$
b_{2n} = \frac{1}{(4\pi)^{d/2}} a_{2n} , \qquad a_{2n} = \alpha_{2n} + \beta_{2n} ,
$$

where α_{2n} and β_{2n} indicate the parts coming from connected and disconnected quantum mechanical diagrams, respectively. In the above expression $\sigma(x)$ is an arbitrary function and $\mathrm{Tr}(\ldots) \equiv \int_M d^d x \sqrt{g}\, \mathrm{tr}_V(\ldots)$. Using the cyclicity of the trace one finds

$$
\beta_0 = 0 , \qquad \beta_2 = 0 , \qquad \beta_4 = \frac{1}{2}\alpha_2^2 , \qquad \beta_6 = \frac{1}{6}\alpha_2^3 + \alpha_2 \alpha_4 .
$$

Then the formulas of [41] for b_0, b_2, b_4, b_6 imply that

$$
\begin{aligned}
\alpha_0 &= 1 \\
\alpha_2 &= E + \frac{1}{6} R \\
\alpha_4 &= \frac{1}{6} \nabla^2 \left(E + \frac{1}{5} R\right) + \frac{1}{180}(R_{abmn}^2 - R_{ab}^2) + \frac{1}{12} F_{ab}^2
\end{aligned}
$$

$$\begin{aligned}
\alpha_6 = \frac{1}{7!}\Big[&18\nabla^4 R + 17(\nabla_a R)^2 - 2(\nabla_a R_{mn})^2 - 4\nabla_a R_{bm}\nabla^b R^{am} + \\
&+ 9(\nabla_a R_{mnij})^2 - 8R_{ab}\nabla^2 R^{ab} + 12 R_{ab}\nabla^a\nabla^b R + 12 R_{abmn}\nabla^2 R^{abmn} + \\
&+ \frac{8}{9}R_a{}^m R_m{}^i R_i{}^a + \frac{8}{3}R_{ab}R_{mn}R^{amnb} - \frac{16}{3}R_{ab}R^a{}_{mnl}R^{bmnl} + \\
&+ \frac{44}{9}R_{ab}{}^{mn}R_{mn}{}^{ij}R_{ij}{}^{ab} - \frac{80}{9}R_{iabj}R^{amnb}R_m{}^{ij}{}_n\Big] + \\
+\frac{2}{6!}\Big[&8(\nabla_a F_{mn})^2 + 2(\nabla^a F_{am})^2 + 12 F_{ab}\nabla^2 F^{ab} - 12 F_a{}^m F_m{}^i F_i{}^a + \\
&+ 6R_{abmn}F^{ab}F^{mn} - 4R_{ab}F^{am}F^b{}_m + 6\nabla^4 E + 30(\nabla_a E)^2 + \\
&+ 4R_{ab}\nabla^a\nabla^b E + 12\nabla_a R\nabla^a E\Big] .
\end{aligned}$$

In terms of the A_s and V_p invariants of appendix A the expression for $b_6 = \frac{1}{(4\pi)^3}(\alpha_6 + \frac{1}{6}\alpha_2^3 + \alpha_2\alpha_4)$ reads as in eq. (2.5).

C. Some b_6 coefficients in arbitrary d

The coefficient $b_6^{(1)}$ of the Hodge-De Rham operator $\Delta^{(1)}$ acting on 1-forms in a d-dimensional manifold is given by

$$\begin{aligned}
b_6^{(1)} = \frac{1}{(4\pi)^{\frac{d}{2}}\,7!}\Bigg(&(18d-84)A_1 + (17d-168)A_2 + (364-2d)A_3 + (56-4d)A_4 + \\
&+ (9d-112)A_5 + (28d-308)A_6 + (840-8d)A_7 + (24d-112)A_8 + \\
&+ (12d-168)A_9 + \left(\frac{35d}{9}-70\right)A_{10} + \left(448-\frac{14d}{3}\right)A_{11} + \\
&+ \left(\frac{14d}{3}-98\right)A_{12} + \left(-\frac{208d}{9}-728\right)A_{13} + \left(\frac{64d}{3}-112\right)A_{14} + \\
&+ \left(476-\frac{16d}{3}\right)A_{15} + \left(\frac{46d}{9}-84\right)A_{16} + \left(\frac{80d}{9}-168\right)A_{17}\Bigg) .
\end{aligned}$$

To compute the coefficient $b_6^{(2)}$ of the Hodge-De Rham operator $\Delta^{(2)}$ acting on 2-forms, we need several relations involving the curvature tensor of the connection and the endomorphism

$$\begin{aligned}
(F_{ij})^{cd}_{ab} &= 2R_{ij[a}{}^{[c}\delta^{d]}_{b]} , \\
E^{cd}_{ab} &= -2R^{[c}_{[a}\delta^{d]}_{b]} + R_{ab}{}^{cd} .
\end{aligned}$$

One can show that

$$\begin{aligned}
\mathrm{tr}\,(F_{ij}F_{kl}) &= (2-d)R_{ijab}R_{klab} \\
\mathrm{tr}\,(F_{ij}F_{kl}F_{mn}) &= (d-2)R_{ijab}R_{klbc}R_{ca} \\
\mathrm{tr}\,E^2 &= R_{ijab}R_{ijab} + (d-6)R_{ab}R_{ab} + R^2
\end{aligned}$$

$$\begin{aligned}\operatorname{tr} E^3 &= R_{ijab}R_{abkl}R_{klij} - 6R_{iabc}R_{jabc}R_{ij} - 6R_{ijab}R_{ja}R_{ib} + \\ &\quad + (10-d)R_{ab}R_{bc}R_{ca} - 3RR_{ab}R_{ab} \\ \operatorname{tr}(EF_{ij}F_{ij}) &= R_{ijab}R_{abkl}R_{klij} + (d-6)R_{iabc}R_{jabc}R_{ij} + RR_{abcd}R_{abcd}\,.\end{aligned}$$

Using these relations, we obtain $b_6^{(2)}$ for an arbitrary d-dimensional manifold

$$\begin{aligned}b_6^{(2)} = \frac{1}{(4\pi)^{\frac{d}{2}}\,7!}\Bigg(&(9d^2-93d+168)A_1 + \left(\frac{17d^2}{2} - \frac{353d}{2} + 756\right)A_2 + \\ &+ (-d^2+365d-2408)A_3 + (-2d^2+58d-112)A_4 + \\ &+ \left(\frac{9d^2}{2} - \frac{233d}{2} + 644\right)A_5 + (14d^2-322d+1456)A_6 + \\ &+ (-4d^2+844d-5040)A_7 + (12d^2-124d+224)A_8 + \\ &+ (6d^2-174d+1176)A_9 + \left(\frac{35d^2}{18} - \frac{1295d}{18} + 560\right)A_{10} + \\ &+ \left(-\frac{7d^2}{3} + \frac{1351d}{3} - 5096\right)A_{11} + \left(\frac{7d^2}{3} - \frac{301d}{3} + 1036\right)A_{12} + \\ &+ \left(\frac{104d^2}{9} - \frac{6448d}{9} + 8176\right)A_{13} + \left(\frac{32d^2}{3} - \frac{368d}{3} + 5264\right)A_{14} + \\ &+ \left(-\frac{8d^2}{3} + \frac{1436d}{3} - 7672\right)A_{15} + \left(\frac{22d^2}{9} - \frac{778d}{9} + 1428\right)A_{16} + \\ &+ \left(\frac{40d^2}{9} - \frac{1552d}{9} + 336\right)A_{17}\Bigg)\,.\end{aligned}$$

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JHEP02(2000)013

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JHEP02(2000)013